



Praktische Lösung des Eigenwertproblems

Unterteilung der Eigenwertlösung in elementare Grundaufgaben

Steigerung von Effizienz und Kondition durch Umformulierung des Eigenwertproblems mit Ähnlichkeitstransformationen:

```
      subroutine rg(nm,n,a,wr,wi,z,iv1,fv1,ierr)
c
c   this subroutine calls the recommended sequence of
c   subroutines from the eigensystem subroutine package (eispack)
c   to find the eigenvalues and eigenvectors
c   of a real general matrix.
c
c   on input
c     nm must be set to the row dimension of the two-dimensional
c     array parameters as declared in the calling program
c     dimension statement.
c     n  is the order of the matrix  a.
c     a  contains the real general matrix.
c   on output
c     wr and wi  contain the real and imaginary parts,
c     respectively, of the eigenvalues.  complex conjugate
c     pairs of eigenvalues appear consecutively with the
c     eigenvalue having the positive imaginary part first.
c     z  contains the real and imaginary parts of the
c     eigenvectors.  if the j-th eigenvalue is real, the
c     j-th column of  z  contains its eigenvector.  if the j-th
c     eigenvalue is complex with positive imaginary part, the
c     j-th and (j+1)-th columns of  z  contain the real and
c     imaginary parts of its eigenvector.  the conjugate of this
c     vector is the eigenvector for the conjugate eigenvalue.
c     ierr is an integer output variable set equal to an error
c     completion code described in the documentation for hqr
c     and hqr2.  the normal completion code is zero.
c     iv1 and fv1 are temporary storage arrays.
c
c   call balanc(nm,n,a,is1,is2,fv1)
c   call elmhes(nm,n,is1,is2,a,iv1)
c   call eltran(nm,n,is1,is2,a,iv1,z)
c   call hqr2(nm,n,is1,is2,a,wr,wi,z,ierr)
c   call balbak(nm,n,is1,is2,fv1,n,z)
c   return
c   end
```



Balancieren

Skalieren des Eigenwertproblems zur Konditionsverbesserung durch Diagonalmatrizen:

```
      subroutine balanc(nm,n,a,low,igh,scale)
c
c   this subroutine balances a real matrix and isolates
c   eigenvalues whenever possible.
c
c   on input
c     n is the order of the matrix.
c     a contains the input matrix to be balanced.
c   on output
c     a contains the balanced matrix.
```

Rücktransformation der Skalierung auf Ursprungsdarstellung nach der Lösung des Eigenwertproblems:

```
      subroutine balbak(nm,n,low,igh,scale,m,z)
c
c   this subroutine forms the eigenvectors of a real general
c   matrix by back transforming those of the corresponding
c   balanced matrix determined by balanc.
c
c   on input
c     n is the order of the matrix.
c     z contains the real and imaginary parts of the eigen-
c     vectors to be back transformed in its first m columns.
c   on output
c     z contains the real and imaginary parts of the
c     transformed eigenvectors in its first m columns.
```



Reduktion auf obere Hessenberg-Form

Transformation auf obere Hessenberg-Form zur Reduktion des Rechenaufwands durch Ähnlichkeitstransformationen:

- 1) Permutationen
- 2) Eliminationen
- 3) Spiegelungen
- 4) Drehungen

```
subroutine elmhes (nm,n,low,igh,a,int)  
c  
c given a real general matrix, this subroutine  
c reduces a submatrix situated in rows and columns  
c low through igh to upper hessenberg form by  
c stabilized elementary similarity transformations.  
c  
c on input  
c n is the order of the matrix.  
c a contains the input matrix.  
c on output  
c a contains the hessenberg matrix. the multipliers  
c which were used in the reduction are stored in the  
c remaining triangle under the hessenberg matrix.  
  
subroutine eltran (nm,n,low,igh,a,int,z)  
c  
c this subroutine accumulates the stabilized elementary  
c similarity transformations used in the reduction of a  
c real general matrix to upper hessenberg form by elmhes.  
c  
c on input  
c n is the order of the matrix.  
c a contains the multipliers which were used in the  
c reduction by elmhes in its lower triangle  
c below the subdiagonal.  
c on output  
c z contains the transformation matrix produced in the  
c reduction by elmhes.
```



Iterative Eigenwertlösung

Überführung des Eigenwertproblems in Rechtsdreiecksform durch

- 1) Jacobi-Verfahren
- 2) LR-Verfahren
- 3) QR-Verfahren

```
subroutine hqr2(nm,n,low,igh,h,wr,wi,z,ierr)
c
c this subroutine finds the eigenvalues and eigenvectors
c of a real upper hessenberg matrix by the qr method. the
c eigenvectors of a real general matrix can also be found
c if elmhes and eltran or orthes and ortran have
c been used to reduce this general matrix to hessenberg form
c and to accumulate the similarity transformations.
c
c on input
c n is the order of the matrix.
c h contains the upper hessenberg matrix.
c z contains the transformation matrix produced by eltran
c after the reduction by elmhes, or by ortran after the
c reduction by orthes, if performed. if the eigenvectors
c of the hessenberg matrix are desired, z must contain the
c identity matrix.
c on output
c wr and wi contain the real and imaginary parts,
c respectively, of the eigenvalues. the eigenvalues
c are unordered except that complex conjugate pairs
c of values appear consecutively with the eigenvalue
c having the positive imaginary part first. if an
c error exit is made, the eigenvalues should be correct
c for indices ierr+1,...,n.
c z contains the real and imaginary parts of the eigenvectors.
c if the i-th eigenvalue is real, the i-th column of z
c contains its eigenvector.
c if the i-th eigenvalue is complex
c with positive imaginary part, the i-th and (i+1)-th
c columns of z contain the real and imaginary parts of its
c eigenvector. the eigenvectors are unnormalized. if an
c error exit is made, none of the eigenvectors
c has been found.
```