Praktische Lösung des Eigenwertproblems

Unterteilung der Eigenwertlösung in elementare Grundaufgaben

Steigerung von Effizienz und Kondition durch Umformulierung des Eigenwertproblems mit Ähnlichkeitstransformationen:

```
      subroutine rg(nm,n,a,wr,wi,z,iv1,fv1,ierr)
c
c     this subroutine calls the recommended sequence of
c     subroutines from the eigensystem subroutine package (eispack)
c     to find the eigenvalues and eigenvectors
c     of a real general matrix.
c
c     on input
c        nm  must be set to the row dimension of the two-dimensional
c        array parameters as declared in the calling program
c        dimension statement.
c        n  is the order of the matrix  a.
c        a  contains the real general matrix.
c     on output
c        wr  and  wi  contain the real and imaginary parts,
c        respectively, of the eigenvalues.  complex conjugate
c        pairs of eigenvalues appear consecutively with the
c        eigenvalue having the positive imaginary part first.
c        z  contains the real and imaginary parts of the
c        eigenvectors. if the j-th eigenvalue is real, the
c        j-th column of  z  contains its eigenvector.  if the j-th
c        eigenvalue is complex with positive imaginary part, the
c        j-th and (j+1)-th columns of  z  contain the real and
c        imaginary parts of its eigenvector.  the conjugate of this
c        vector is the eigenvector for the conjugate eigenvalue.
c        ierr  is an integer output variable set equal to an error
c           completion code described in the documentation for hqr
c           and hqr2.  the normal completion code is zero.
c        iv1  and  fv1  are temporary storage arrays.
c
      call balanc(nm,n,a,is1,is2,fv1)
      call elmhes(nm,n,is1,is2,a,iv1)
      call eltran(nm,n,is1,is2,a,iv1,z)
      call hqr2(nm,n,is1,is2,a,wr,wi,z,ierr)
      call balbak(nm,n,is1,is2,fv1,n,z)
      return
      end
```

Balancieren

Skalieren des Eigenwertproblems zur Konditionsverbesserung durch Diagonalmatrizen:

```
      subroutine balanc(nm,n,a,low,igh,scale)
c
c     this subroutine balances a real matrix and isolates
c     eigenvalues whenever possible.
c
c     on input
c        n is the order of the matrix.
c        a contains the input matrix to be balanced.
c     on output
c        a contains the balanced matrix.
```

Rücktransformation der Skalierung auf Ursprungsdarstellung nach der Lösung des Eigenwertproblems:

```
      subroutine balbak(nm,n,low,igh,scale,m,z)
c
c     this subroutine forms the eigenvectors of a real general
c     matrix by back transforming those of the corresponding
c     balanced matrix determined by  balanc.
c
c     on input
c        n is the order of the matrix.
c        z  contains the real and imaginary parts of the eigen-
c          vectors to be back transformed in its first m columns.
c     on output
c        z contains the real and imaginary parts of the
c          transformed eigenvectors in its first m columns.
```

Reduktion auf obere Hessenberg–Form

Transformation auf obere Hessenberg–Form zur Reduktion des Rechenaufwands durch Ähnlichkeitstransformationen:

1) Permutationen
2) Eliminationen
3) Spiegelungen
4) Drehungen

```
      subroutine elmhes(nm,n,low,igh,a,int)
c
c     given a real general matrix, this subroutine
c     reduces a submatrix situated in rows and columns
c     low through igh to upper hessenberg form by
c     stabilized elementary similarity transformations.
c
c     on input
c        n is the order of the matrix.
c        a contains the input matrix.
c     on output
c        a contains the hessenberg matrix.  the multipliers
c          which were used in the reduction are stored in the
c          remaining triangle under the hessenberg matrix.
```

```
      subroutine eltran(nm,n,low,igh,a,int,z)
c
c     this subroutine accumulates the stabilized elementary
c     similarity transformations used in the reduction of a
c     real general matrix to upper hessenberg form by  elmhes.
c
c     on input
c        n is the order of the matrix.
c        a contains the multipliers which were used in the
c          reduction by  elmhes  in its lower triangle
c          below the subdiagonal.
c     on output
c        z contains the transformation matrix produced in the
c          reduction by  elmhes.
```

Iterative Eigenwertlösung

Überführung des Eigenwertproblems in Rechtsdreiecksform durch

1) Jacobi–Verfahren
2) LR–Verfahren
3) QR–Verfahren

```
      subroutine hqr2(nm,n,low,igh,h,wr,wi,z,ierr)
c
c     this subroutine finds the eigenvalues and eigenvectors
c     of a real upper hessenberg matrix by the qr method.  the
c     eigenvectors of a real general matrix can also be found
c     if  elmhes  and  eltran  or  orthes  and  ortran  have
c     been used to reduce this general matrix to hessenberg form
c     and to accumulate the similarity transformations.
c
c     on input
c        n is the order of the matrix.
c        h contains the upper hessenberg matrix.
c        z contains the transformation matrix produced by  eltran
c          after the reduction by  elmhes, or by  ortran  after the
c          reduction by  orthes, if performed.  if the eigenvectors
c          of the hessenberg matrix are desired, z must contain the
c          identity matrix.
c     on output
c        wr and wi contain the real and imaginary parts,
c          respectively, of the eigenvalues.  the eigenvalues
c          are unordered except that complex conjugate pairs
c          of values appear consecutively with the eigenvalue
c          having the positive imaginary part first.  if an
c          error exit is made, the eigenvalues should be correct
c          for indices ierr+1,...,n.
c        z contains the real and imaginary parts of the eigenvectors.
c          if the i-th eigenvalue is real, the i-th column of z
c          contains its eigenvector.
c          if the i-th eigenvalue is complex
c          with positive imaginary part, the i-th and (i+1)-th
c          columns of z contain the real and imaginary parts of its
c          eigenvector.  the eigenvectors are unnormalized.  if an
c          error exit is made, none of the eigenvectors
c          has been found.
```