Determinanten

Die Determinante einer quadratischen, reellen Matrix $A = [\boldsymbol{a}_1 \dots \boldsymbol{a}_n] \in \mathbb{R}^{n\times n}$ ist durch folgende Grundgesetze definiert:

(1) $\det[\boldsymbol{a}_1 \dots \boldsymbol{a}_i + \boldsymbol{b}_i \dots \boldsymbol{a}_n] = \det[\boldsymbol{a}_1 \dots \boldsymbol{a}_i \dots \boldsymbol{a}_n] + \det[\boldsymbol{a}_1 \dots \boldsymbol{b}_i \dots \boldsymbol{a}_n]$

(2) $\det[\boldsymbol{a}_1 \dots \lambda\boldsymbol{a}_i \dots \boldsymbol{a}_n] = \lambda\det[\boldsymbol{a}_1 \dots \boldsymbol{a}_i \dots \boldsymbol{a}_n]$

(3) $\det\big[\dots \boldsymbol{a}_i \dots \boldsymbol{a}_j \dots\big] = -\det[\dots \boldsymbol{a}_j \dots \boldsymbol{a}_i \dots]$

(4) $\det \boldsymbol{E} = 1$

Determinantenformel:

$$\begin{aligned}
\det \boldsymbol{A} &= \det[\boldsymbol{a}_1 \dots \boldsymbol{a}_n] \\
&= \det[a_{11}\boldsymbol{e}_1 + \dots + a_{n1}\boldsymbol{e}_n, \dots, a_{1n}\boldsymbol{e}_1 + \dots + a_{nn}\boldsymbol{e}_n] \\
&= \sum_{i_1,\dots,i_n \in \{1,\dots,n\}} a_{i_1 1} a_{i_2 2} \dots a_{i_n n} \det[e_{i_1} \dots e_{i_n}] \\
&= \sum_{\substack{(i_1 \dots i_n) \\ \text{Permutation von } (1\dots n)}} \operatorname{sign}(i_1 \dots i_n) a_{i_1 1} a_{i_2 2} \dots a_{i_n n}
\end{aligned}$$

Rechenregeln:

- $\det[\dots \boldsymbol{a} \dots \boldsymbol{a} \dots] = 0$
- $\det[\boldsymbol{a}_1 \dots \boldsymbol{0} \dots \boldsymbol{a}_n] = 0$
- $\det\big[\dots \boldsymbol{a}_i \dots \boldsymbol{a}_j + \lambda\boldsymbol{a}_i \dots\big] = \det[\dots \boldsymbol{a}_i \dots \boldsymbol{a}_j \dots]$
- $\det \boldsymbol{A}^T = \det \boldsymbol{A}$
- $\det \boldsymbol{A}^{-1} = \frac{1}{\det \boldsymbol{A}}$
- $\det \boldsymbol{L} = \prod_{i=1}^n L_{ii}, \quad \det \boldsymbol{R} = \prod_{i=1}^n R_{ii}$
- $\det \boldsymbol{A} \neq 0 \Leftrightarrow \boldsymbol{A}$ regulär
- $\det(\boldsymbol{A} \cdot \boldsymbol{B}) = \det(\boldsymbol{A}) \det(\boldsymbol{B})$