Schwinger mit einem Freiheitsgrad

Schwingungsgleichung: $\ddot{y}(t) + 2\,\delta\,\dot{y}(t) + \omega_0^2\,y(t) = 0$

Anfangsbedingungen: Lage $y(0) = y_0$

Geschwindigkeit $\dot{y}(0) = \dot{y}_0$

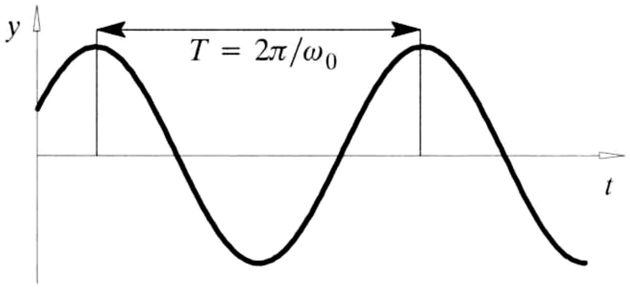
Lösungsansatz: $y(t) = e^{\lambda t}$ $\Rightarrow y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t},\ C_1, C_2 \in \mathbb{C}$

$$\lambda_{1,2} = -\delta \pm \sqrt{\delta^2 - \omega_0^2}$$

Diskussion der Eigenwerte

<table>
<tr><th colspan="2">Fallunterscheidung</th><th>Eigenwerte</th><th>Schwingung</th></tr>
<tr><td>keine Dämpfung</td><td>$\delta = 0$</td><td>$\lambda_{1,2} = \pm i\,\omega_0$</td><td>

$$\begin{aligned} y(t) &= C_1 e^{i\omega_0 t} + C_2 e^{-i\omega_0 t} \\ &= (C_1 + C_2)\cos\omega_0 t + (C_1 - C_2) i \sin\omega_0 t \\ &= A\sin\omega_0 t + B\cos\omega_0 t \\ &= C\cos(\omega_0 t - \varphi) \end{aligned}$$



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<tr><td>schwache Dämpfung</td><td>$\delta^2 < \omega_0^2$</td><td>

$\lambda_{1,2} = -\delta \pm i\omega$

$\omega = \sqrt{\omega_0^2 - \delta^2}$

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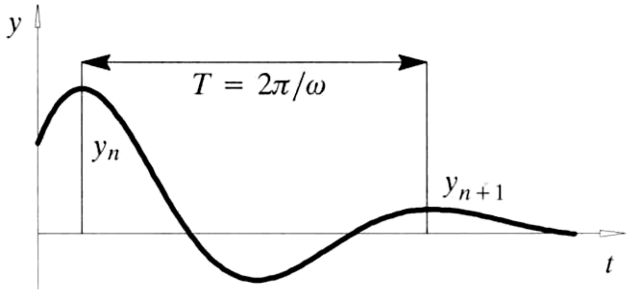
$$\begin{aligned} y(t) &= C_1 e^{(-\delta + i\omega)t} + C_2 e^{(-\delta - i\omega)t} \\ &= e^{-\delta t}(C_1 e^{i\omega t} + C_2 e^{-i\omega t}) \\ &= e^{-\delta t}(A\sin\omega t + B\cos\omega t) \\ &= Ce^{-\delta t}\cos(\omega t - \varphi) \end{aligned}$$



log. Dekrement $\vartheta = \delta T = \ln\frac{y_n}{y_{n+1}}$

Lehrsches Dämpfungsmaß $D = \frac{\delta}{\omega_0} = \frac{\vartheta}{\sqrt{\vartheta^2 + 4\pi^2}}$

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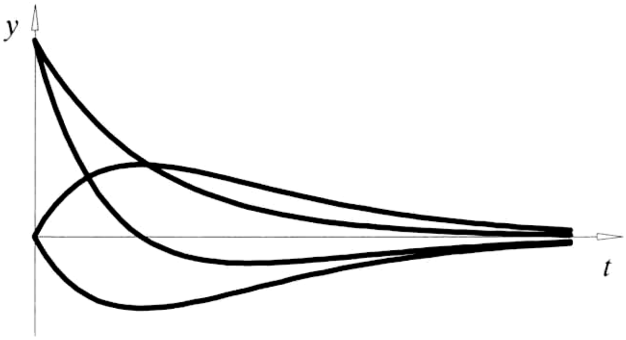
Grenzfall	$\delta^2 = \omega_0^2$	$\lambda_{1,2} = -\delta$	$y(t) = e^{-\delta t}(\bar{A}t + \bar{B})$ 
starke Dämpfung	$\delta^2 > \omega_0^2$	$\lambda_1 \neq \lambda_2 \in \mathbb{R}^-$	$y(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$