Standarddatenberechnung in Flexiblen Mehrkörpersystemen

Massenmatrix

$$\mathbf{M}(\mathbf{q}) = \begin{bmatrix} \int_{\Omega_0} \mathbf{E}\,\mathrm{dm} & \int_{\Omega_0} \tilde{\mathbf{r}}_{\mathrm{RP}}^{\mathrm{T}}\,\mathrm{dm} & \int_{\Omega_0} \mathbf{\Phi}\,\mathrm{dm} \\ & \int_{\Omega_0} \tilde{\mathbf{r}}_{\mathrm{RP}}\tilde{\mathbf{r}}_{\mathrm{RP}}^{\mathrm{T}}\,\mathrm{dm} & \int_{\Omega_0} \tilde{\mathbf{r}}_{\mathrm{RP}}\mathbf{\Phi}\,\mathrm{dm} \\ \mathrm{sym.} & & \int_{\Omega_0} \mathbf{\Phi}^{\mathrm{T}}\mathbf{\Phi}\,\mathrm{dm} \end{bmatrix}$$

Masse

$$\int_{\Omega_0} \mathbf{E}\,\mathrm{dm} = \begin{bmatrix} \mathrm{m} & 0 & 0 \\ 0 & \mathrm{m} & 0 \\ 0 & 0 & \mathrm{m} \end{bmatrix}$$

Schwerpunkt

$$\mathrm{m}\tilde{\mathbf{c}}^{\mathrm{T}} = \int_{\Omega_0} \tilde{\mathbf{r}}_{\mathrm{RP}}^{\mathrm{T}}\,\mathrm{dm} = \mathrm{m}\left(\widetilde{\mathbf{c}_0 + \mathbf{c}_1(\mathbf{q})}\right)^{\mathrm{T}}$$

$$\mathbf{c}_0 = \frac{1}{\mathrm{m}} \int_{\Omega_0} \mathbf{R}_{\mathrm{RP}}\,\mathrm{dm}$$

$$\mathbf{c}_1(\mathbf{q}) = \frac{1}{\mathrm{m}} \int_{\Omega_0} \mathbf{\Phi}\,\mathrm{dm}\,\mathbf{q} = \frac{1}{\mathrm{m}}\mathbf{C1}\,\mathbf{q}$$

Trägheitstensor

$$\mathbf{I}(\mathbf{q}) = \int_{\Omega_0} \tilde{\mathbf{r}}_{\mathrm{RP}}\tilde{\mathbf{r}}_{\mathrm{RP}}^{\mathrm{T}}\,\mathrm{dm} = \mathbf{I}_0 + \mathbf{I}_1(\mathbf{q}) + \mathbf{I}_2(\mathbf{q})$$

$$\mathbf{I}_0 = \int_{\Omega_0} \tilde{\mathbf{R}}_{\mathrm{RP}}\tilde{\mathbf{R}}_{\mathrm{RP}}^{\mathrm{T}}\,\mathrm{dm}$$

$$\mathbf{I}_1(\mathbf{q}) = \int_{\Omega_0} \left(\tilde{\mathbf{R}}_{\mathrm{RP}}(\widetilde{\mathbf{\Phi q}})^{\mathrm{T}} + (\widetilde{\mathbf{\Phi q}})\tilde{\mathbf{R}}_{\mathrm{RP}}^{\mathrm{T}}\right)\mathrm{dm} = -\sum_{\mathrm{l}=1}^{\mathrm{n_q}} \left(\mathbf{C4}_{\mathrm{l}} + \mathbf{C4}_{\mathrm{l}}^{\mathrm{T}}\right)\mathrm{q_l}$$

$$\mathbf{I}_2(\mathbf{q}) = \int_{\Omega_0} (\widetilde{\mathbf{\Phi q}})(\widetilde{\mathbf{\Phi q}})^{\mathrm{T}}\,\mathrm{dm}$$

($\mathbf{I}_2$ entfällt bei Linearisierung in den Verformungskoordinaten)

Kopplungsterme - Translation

$$\mathbf{C}_t = \int_{\Omega_0} \mathbf{\Phi}^T \,\mathrm{dm} = \mathbf{C1}^T$$

Kopplungsterme - Rotation

$$\mathbf{C}_r(\mathbf{q}) = \int_{\Omega_0} \mathbf{\Phi}^T \tilde{\mathbf{r}}_{RP}^T \,\mathrm{dm} = \mathbf{C}_{r0} + \mathbf{C}_{r1}(\mathbf{q})$$

$$\mathbf{C}_{r0} = \int_{\Omega_0} \mathbf{\Phi}^T \tilde{\mathbf{R}}_{RP}^T \,\mathrm{dm} = \mathbf{C2}^T$$

$$\mathbf{C}_{r1}(\mathbf{q}) = \int_{\Omega_0} \mathbf{\Phi}^T (\widetilde{\mathbf{\Phi}\mathbf{q}})^T \,\mathrm{dm} = \sum_{l=1}^{n_q} \mathbf{C5}_l^T q_l$$

Massenmatrix des elastischen Subsystems

$$\mathbf{M}_e = \int_{\Omega_0} \mathbf{\Phi}^T \mathbf{\Phi} \,\mathrm{dm} = \mathbf{C3}_{11} + \mathbf{C3}_{22} + \mathbf{C3}_{33}$$

Generalisierte Trägheitskräfte

$$\begin{bmatrix} \mathbf{h}_{\omega t} \\ \mathbf{h}_{\omega r} \\ \mathbf{h}_{\omega e} \end{bmatrix} = \begin{bmatrix} \int_{\Omega_0} \tilde{\boldsymbol{\omega}}_{IR} \tilde{\boldsymbol{\omega}}_{IR} \mathbf{r}_{RP} + 2 \tilde{\boldsymbol{\omega}}_{IR} \dot{\mathbf{r}}_{RP} \,\mathrm{dm} \\ -\int_{\Omega_0} \tilde{\mathbf{r}}_{RP} \tilde{\boldsymbol{\omega}}_{IR} \tilde{\boldsymbol{\omega}}_{IR} \mathbf{r}_{RP} + 2 \tilde{\mathbf{r}}_{RP} \tilde{\boldsymbol{\omega}}_{IR} \dot{\mathbf{r}}_{RP} \,\mathrm{dm} \\ \int_{\Omega_0} \mathbf{\Phi}^T \tilde{\boldsymbol{\omega}}_{IR} \tilde{\boldsymbol{\omega}}_{IR} \mathbf{r}_{RP} + 2 \mathbf{\Phi}^T \tilde{\boldsymbol{\omega}}_{IR} \dot{\mathbf{r}}_{RP} \,\mathrm{dm} \end{bmatrix}$$

Anteile aus der Translation

$$\mathbf{h}_{\omega t} = 2\, \tilde{\boldsymbol{\omega}}_{IR} \mathbf{C1}\, \dot{\mathbf{q}} + m\, \tilde{\boldsymbol{\omega}}_{IR} \tilde{\boldsymbol{\omega}}_{IR} \mathbf{c}$$

Anteile aus der Rotation

$$\mathbf{h}_{\omega r} = \tilde{\boldsymbol{\omega}}_{IR} \mathbf{I} \boldsymbol{\omega}_{IR} + \sum_{l=1}^{n_q} \mathbf{G}_{rl}(\mathbf{q}) \dot{q}_l \boldsymbol{\omega}_{IR}$$

$$\mathbf{G}_{rl}(\mathbf{q}) = -2\, \mathbf{C4}_l - 2 \sum_{k=1}^{n_q} \mathbf{C6}_{kl} q_k$$

Anteile aus den elastischen Verformungen

$$\mathbf{h}_{\omega e} = \begin{bmatrix} \boldsymbol{\omega}_{IR}^T \mathbf{O}_e^1 \boldsymbol{\omega}_{IR} \\ \vdots \\ \boldsymbol{\omega}_{IR}^T \mathbf{O}_e^{n_q} \boldsymbol{\omega}_{IR} \end{bmatrix} + \sum_{l=1}^{n_q} \mathbf{G}_{el} \dot{q}_l \boldsymbol{\omega}_{IR}$$

$$\mathbf{O}_e^k = \int_{\Omega_0} \left(\tilde{\mathbf{R}} \tilde{\boldsymbol{\Phi}}_{*k} \right)^T \mathrm{d}m + \sum_{l=1}^{n_q} \int_{\Omega_0} \tilde{\boldsymbol{\Phi}}_{*k} \tilde{\boldsymbol{\Phi}}_{*l} \, \mathrm{d}m \, q_l = \mathbf{C4}_k^T + \sum_{l=1}^{n_q} \mathbf{C6}_{kl} q_l$$

$$\mathbf{G}_{el} = 2 \int_{\Omega_0} \left(\tilde{\boldsymbol{\Phi}}_{*l} \boldsymbol{\Phi} \right)^T \mathrm{d}m = 2 \, \mathbf{C5}_l^T$$

Neben der Berechnung von m, $\mathbf{c}_0$ und $\mathbf{I}_0$ müssen die folgenden Integrale für eine effiziente Auswertung der Bewegungsgleichungen im Vorfeld der Zeitintegration berechnet werden:

Integral	Dimension
$\mathbf{C1} = \int_{\Omega_0} \boldsymbol{\Phi} \, \mathrm{d}m$	$[3 \times n_q]$
$\mathbf{C2} = \int_{\Omega_0} \tilde{\mathbf{R}} \boldsymbol{\Phi} \, \mathrm{d}m$	$[3 \times n_q]$
$\mathbf{C3}_{\alpha\beta} = \int_{\Omega_0} \boldsymbol{\Phi}_{\alpha *}^T \boldsymbol{\Phi}_{\beta *} \, \mathrm{d}m$	$[n_q \times n_q]$
$\mathbf{C4}_l = \int_{\Omega_0} \tilde{\mathbf{R}} \tilde{\boldsymbol{\Phi}}_{*l} \, \mathrm{d}m$	$[3 \times 3]$
$\mathbf{C5}_l = \int_{\Omega_0} \tilde{\boldsymbol{\Phi}}_{*l} \boldsymbol{\Phi} \, \mathrm{d}m$	$[3 \times n_q]$
$\mathbf{C6}_{kl} = \int_{\Omega_0} \tilde{\boldsymbol{\Phi}}_{*k} \tilde{\boldsymbol{\Phi}}_{*l} \, \mathrm{d}m$	$[3 \times 3]$

Hinweise zur Notation:

$l, k = 1, \dots n_q$

$\alpha, \beta = 1, 2, 3$

$\boldsymbol{\Phi}_{\alpha *}$: α-te Zeile von $\boldsymbol{\Phi}$

$\boldsymbol{\Phi}_{*l}$: l-te Spalte von $\boldsymbol{\Phi}$