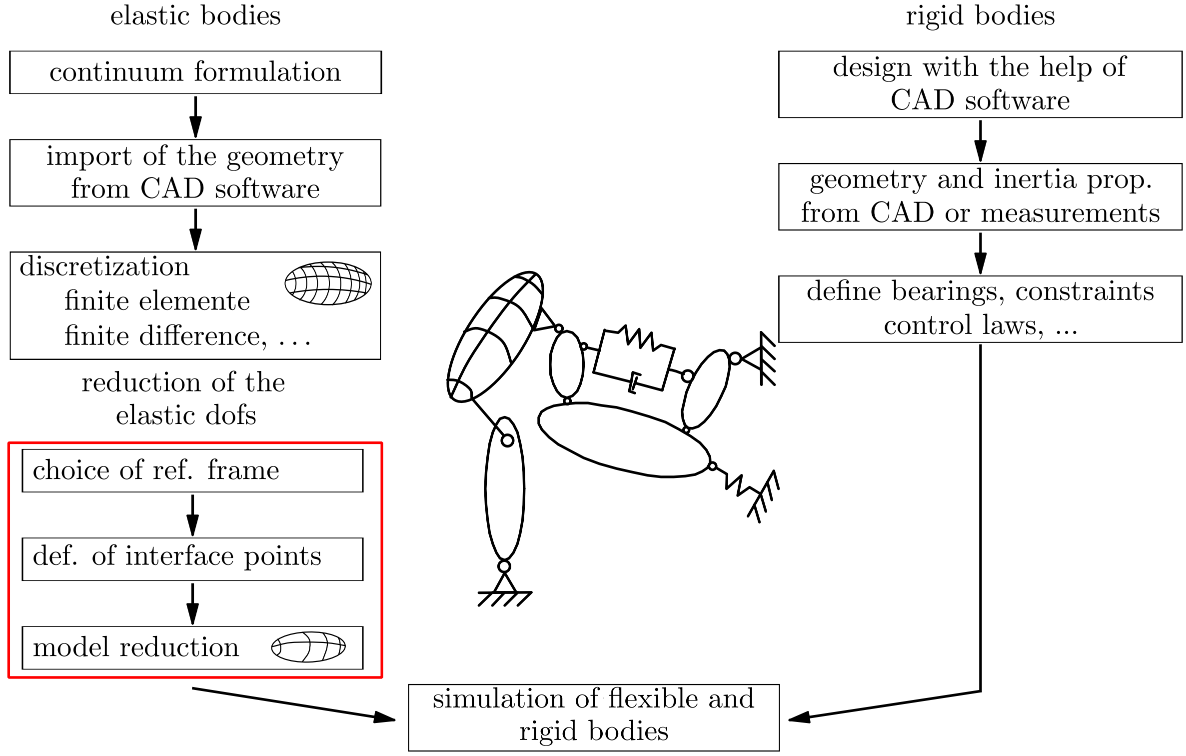


Figure 1: Preprocessing for the simulation of a flexible multibody system

Extracted from Doctoral Thesis Jörg Fehr [1].

# Workflow of Model Reduction in Elastic Multipbody Systems

The workflow of the simulation of an elastic is shown in Fig. 1.

# 1 Elastic Multibody Systems with the Floating Frame of Reference Formulation

In a floating frame of reference formulation, the motion $\boldsymbol{r}(\boldsymbol{R}, t)$ of a point $P$ of an elastic body is separated into a usually nonlinear motion of the reference frame $\boldsymbol{K}_R$ and into a motion with respect to the reference frame $\boldsymbol{r}_{RP}$, see Figure 2.

## 1.1 Kinematics of Elastic Multibody Systems

In the deformed state at time $t$ the position $\boldsymbol{r}_P(t)$ of a point $P$ is expressed as

$$\boldsymbol{r}_P(t) = \boldsymbol{r}_{IR}(t) + \boldsymbol{R}_{RP} + \boldsymbol{u}_P(t) \tag{1}$$

with the nonlinear motion $\boldsymbol{r}_{IR}(t)$ of the reference frame $\boldsymbol{K}_R$, the position of point $P$ in the undeformed state $\boldsymbol{R}_{RP}$ and an elastic deformation $\boldsymbol{u}_P(t)$ which is measured with respect to the motion of the reference frame. In the following, only one elastic body will be considered. A rigid body can be

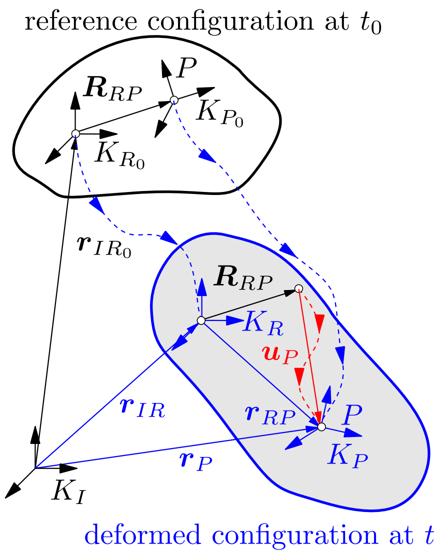


Figure 2: Floating frame of reference formulation

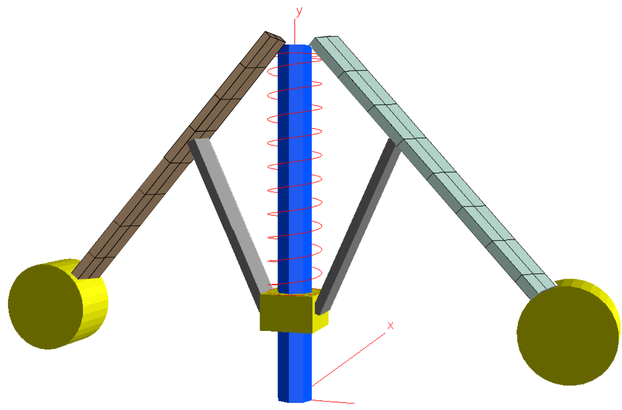

Figure 3: Simpack model of an elastic multibody system (governor controller with two flexible arms)

treated as a special case of elastic bodies where the displacement field $\boldsymbol{u}$ is zero. The rotation-matrix $\boldsymbol{A}_{RP}$ transforms the vector ${}_P\boldsymbol{v}$ expressed in the frame $\boldsymbol{K}_P$ into the vector ${}_i\boldsymbol{v}$ in the reference frame $\boldsymbol{K}_R$. With three independent variables $\vartheta_{iP}$ the orthogonal rotation matrix $\boldsymbol{A}_{RP}$ can be derived as shown e.g. in [2]. Different choices of variables are possible, e.g. Euler or Cardan Angles or Euler Parameters. Similarly to the position $\boldsymbol{r}_P(t)$, the transformation matrix $\boldsymbol{A}_{IP}$ is split in two transformation matrices

$$\boldsymbol{A}_{IP}(t) = \boldsymbol{A}_{IR}(t) \cdot \boldsymbol{A}_{RP}(t) \tag{2}$$

in which $\boldsymbol{A}_{IR}(t)$ defines a coordinate transformation from frame $\boldsymbol{K}_R$ to frame $\boldsymbol{K}_I$. Here, only small deformations are considered and the transformation matrix $\boldsymbol{A}_{RP}(t)$ from frame $\boldsymbol{K}_P$ to frame $\boldsymbol{K}_R$ is divided into a constant part $\boldsymbol{\Gamma}_{RP}$ and a time dependent part $\boldsymbol{I} + \tilde{\boldsymbol{\vartheta}}_P(t)$

$$\boldsymbol{A}_{RP}(t) = \boldsymbol{\Gamma}_{RP} \cdot (\boldsymbol{I} + \tilde{\boldsymbol{\vartheta}}_P(t)), \tag{3}$$

where $\tilde{\boldsymbol{\vartheta}}_P(t)$ is the skew-symmetric matrix of the rotational angles describing the orientation of a frame attached to point $P$ collected in the rotation vector $\boldsymbol{\vartheta}_P(t)$.

Two global Rayleigh-Ritz approaches are introduced to approximate the elastic deformation $\boldsymbol{u}(\boldsymbol{R}, t) = \boldsymbol{\Phi}(\boldsymbol{R}) \cdot \boldsymbol{q}(t)$ and the small rotations $\boldsymbol{\vartheta}(\boldsymbol{R}, t) = \boldsymbol{\Psi}(\boldsymbol{R}) \cdot \boldsymbol{q}(t)$ of the body. According to [3], the ansatz functions have to be at least elements of a complete function space, they have to be sufficiently smooth and the geometric boundary conditions have to be fulfilled. A systematic way to find suitable ansatz functions is the application of the FE shape functions $\bar{\boldsymbol{N}}(\boldsymbol{R})$ constrained by the boundary conditions. In a second step, the FE ansatz functions are reduced by a reduction method which extracts the dominant shape functions of the system. Concerning beam and plate elements, the orientation of a frame attached to an FE node can be expressed by the nodal rotation parameters of the nodes and the rotational ansatz functions $\boldsymbol{\Psi}$ are part of the global FE shape functions $\bar{\boldsymbol{N}}(\boldsymbol{R})$. However, for Lagrangian elements with only nodal displacements as degrees of freedom, the orientation of a frame attached to a material point $P$ is not explicitly considered but the information about rotations is included in the displacement field, see e.g. [4, 5]. Hence, it is possible to calculate the orientation of a frame in the deformed state [6]. In [3, 7] it is discussed that Cartesian coordinate systems in the undeformed state are no longer Cartesian coordinate systems in the deformed state. In order to calculate rotational ansatz functions for nodes connected with Lagrangian elements the approach suggested in [8, 9] is used. This approach defines a rigid region around the nodes and reconstructs the rotational ansatz functions from the position of three points in the rigid region. As to point $P$ the elastic deformation $\boldsymbol{u}_P(t)$ and the small rotations $\boldsymbol{\vartheta}_P(t)$ are now expressed as

$$\boldsymbol{u}_P(t) = \boldsymbol{\Phi}(\boldsymbol{R}_{RP}) \cdot \boldsymbol{q}(t) = \boldsymbol{\Phi}_P \cdot \boldsymbol{q}(t), \qquad \boldsymbol{\vartheta}_P(t) = \boldsymbol{\Psi}(\boldsymbol{R}_{RP}) \cdot \boldsymbol{q}(t) = \boldsymbol{\Psi}_P \cdot \boldsymbol{q}(t). \tag{4}$$

The absolute motion of a point of an arbitrary frame expressed in the referential frame is obtained by total differentiation and reads

$$\text{position} \qquad \boldsymbol{r}_{IP}(t) = \boldsymbol{r}_{IR}(t_0) + \boldsymbol{R}_{RP} + \boldsymbol{\Phi}_P \cdot \boldsymbol{q}(t), \tag{5}$$

$$\text{orientation} \qquad \boldsymbol{A}_{IP}(\boldsymbol{R}, t) = \boldsymbol{A}_{Ii}(t) \cdot \boldsymbol{\Gamma}_{RP} \cdot (\boldsymbol{I} + (\widetilde{\boldsymbol{\Psi}_P \cdot \boldsymbol{q}(t)})), \tag{6}$$

$$\text{velocity} \qquad \boldsymbol{v}_{IP} = \underbrace{\begin{bmatrix} \boldsymbol{I} & -\widetilde{\boldsymbol{R}}_{RP} & \boldsymbol{\Phi}_P \end{bmatrix}}_{\boldsymbol{T}^t_{RP}} \cdot \begin{bmatrix} \mathbf{v}_{IR} \\ \boldsymbol{\omega}_{IR} \\ \dot{\boldsymbol{q}} \end{bmatrix}, \tag{7}$$

$$\text{angular velocity} \qquad \boldsymbol{\omega}_{IP} = \underbrace{\begin{bmatrix} \mathbf{0} & \boldsymbol{I} & \boldsymbol{\Psi}_P \end{bmatrix}}_{\boldsymbol{T}^r_{RP}} \cdot \begin{bmatrix} \boldsymbol{v}_{IR} \\ \boldsymbol{\omega}_{IR} \\ \dot{\boldsymbol{q}}^P \end{bmatrix}, \tag{8}$$

$$\begin{aligned} \text{acceleration} \qquad \boldsymbol{a}_{IP} &= \underbrace{\begin{bmatrix} \boldsymbol{I} & -\widetilde{\boldsymbol{R}}_{RP} & \boldsymbol{\Phi}_P \end{bmatrix}}_{\boldsymbol{T}^t_{RP}} \cdot \begin{bmatrix} \boldsymbol{a}_{IR} \\ \boldsymbol{\alpha}_{IR} \\ \ddot{\boldsymbol{q}} \end{bmatrix} \\ &\quad + \underbrace{\widetilde{\boldsymbol{\omega}}_{IR} \cdot \widetilde{\boldsymbol{\omega}}_{IR} \cdot \boldsymbol{r}_{RP} + 2\widetilde{\boldsymbol{\omega}}_{IR} \cdot \dot{\boldsymbol{r}}_{RP}}_{\zeta^t_{RP}}, \end{aligned} \tag{9}$$

$$\text{angular acceleration} \qquad \boldsymbol{\alpha}_{IP} = \underbrace{\begin{bmatrix} \mathbf{0} & \boldsymbol{I} & \boldsymbol{\Psi}_p \end{bmatrix}}_{\boldsymbol{T}^r_{RP}} \cdot \begin{bmatrix} \boldsymbol{a}_{IR} \\ \boldsymbol{\alpha}_{IR} \\ \ddot{\boldsymbol{q}} \end{bmatrix} + \underbrace{\widetilde{\boldsymbol{\omega}}_{IR} \cdot \boldsymbol{\omega}_{RP}}_{\zeta^r_{RP}}, \tag{10}$$

with the vector of angular velocity of frame $K_R$ which can be calculated from

$$\widetilde{\boldsymbol{\omega}}_{IR} = \boldsymbol{A}^T_{IR} \cdot \dot{\boldsymbol{A}}_{IR} \tag{11}$$

and the angular acceleration $\boldsymbol{\alpha}_{IR} = \dot{\boldsymbol{\omega}}_{IR}$.

## 1.2 Kinetics of a Deformable Body

By using principles of dynamics, the equation of motion of the body can be derived. Jourdain's principle of dynamics, also known as the principle of virtual power, is used in [3, 9] for the derivation of the equation of motion. As an alternative, the principle of virtual work can be used for the derivation of the equation of motion [10]. The generalized Newton-Euler equation of the body reads

$$\begin{bmatrix} m\boldsymbol{I} & m\widetilde{\boldsymbol{c}}^T(\boldsymbol{q}) & \boldsymbol{C}_t^T \\ m\widetilde{\boldsymbol{c}}(\boldsymbol{q}) & \boldsymbol{J}(\boldsymbol{q}) & \boldsymbol{C}_r^T(\boldsymbol{q}) \\ \boldsymbol{C}_t & \boldsymbol{C}_r(\boldsymbol{q}) & \boldsymbol{M}_e \end{bmatrix} \cdot \begin{bmatrix} \dot{\boldsymbol{v}}_{IR} \\ \dot{\boldsymbol{\omega}}_{IR} \\ \ddot{\boldsymbol{q}} \end{bmatrix} = -\underbrace{\begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{k}_\sigma + (\boldsymbol{K}_{eL} + \boldsymbol{K}_{eN}(\boldsymbol{q})) \cdot \boldsymbol{q} + \boldsymbol{D}_e \cdot \dot{\boldsymbol{q}} \end{bmatrix}}_{\boldsymbol{h}_e(\boldsymbol{q},\dot{\boldsymbol{q}}) \text{ internal forces}}$$
$$-\underbrace{\begin{bmatrix} m\widetilde{\boldsymbol{\omega}}_{IR} \cdot \dot{\boldsymbol{v}}_{IR} + m\widetilde{\boldsymbol{\omega}}_{IR} \cdot \widetilde{\boldsymbol{\omega}}_{IR} \cdot \boldsymbol{c} + 2m\boldsymbol{\omega}_{IR} \cdot \dot{\boldsymbol{c}}(\boldsymbol{q}) \\ m\widetilde{\boldsymbol{c}}(\boldsymbol{q}) \cdot \widetilde{\boldsymbol{\omega}}_{IR} \cdot \dot{\boldsymbol{v}}_{IR} + (\sum_{l=1}^N \boldsymbol{G}_{rl}(\boldsymbol{q},\dot{\boldsymbol{q}}) \cdot \dot{q}_l) \cdot \boldsymbol{\omega}_{IR} + \widetilde{\boldsymbol{\omega}}_{IR} \cdot \boldsymbol{J}(\boldsymbol{q}) \cdot \boldsymbol{\omega}_{IR} \\ \boldsymbol{C}_t \cdot \boldsymbol{\omega}_{IR} \cdot \boldsymbol{v}_{IR} + (\sum_{l=1}^N \boldsymbol{G}_{el}(\dot{\boldsymbol{q}}) \cdot \dot{q}_l) \cdot \boldsymbol{\omega}_{IR} + \boldsymbol{O}_{eP}(\boldsymbol{q}) \end{bmatrix}}_{\boldsymbol{h}_\omega \text{ inertia forces}} + \underbrace{\begin{bmatrix} m\boldsymbol{I} \\ m\widetilde{\boldsymbol{c}} \\ \boldsymbol{C}_t \end{bmatrix} \cdot \boldsymbol{g}}_{\boldsymbol{h}_g}$$
$$+ \underbrace{\int_{\Gamma_p} \begin{bmatrix} \boldsymbol{I} \\ \widetilde{\boldsymbol{r}}_{RP} \\ \boldsymbol{\Phi}(\boldsymbol{R})^T \end{bmatrix} \cdot \boldsymbol{p}_b dA}_{\boldsymbol{h}_P \text{ ext. surface loads}} + \underbrace{\sum_k \left( \begin{bmatrix} \boldsymbol{I} \\ \widetilde{\boldsymbol{r}}_{PRk} \\ \boldsymbol{\Phi}_k^T \end{bmatrix} \cdot \boldsymbol{F}_k + \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{I} \\ \boldsymbol{\Psi}_k^T \end{bmatrix} \cdot \boldsymbol{L}_k \right)}_{\boldsymbol{h}_d \text{ ext. point forces}} \tag{12}$$

with the mass $m$ of the body and the $3 \times 3$ inertia tensor $\boldsymbol{J}$ in its deformed configuration. The location of the center of mass $\boldsymbol{c}$ and the elastic mass matrix $\boldsymbol{M}_e \in \mathbb{R}^{N \times N}$, the matrices $\boldsymbol{C}_t$ and $\boldsymbol{C}_r$ represent the coupling between the reference motion and deformations. The internal forces result from the FE equation plus the vector $\boldsymbol{k}_\sigma$ which represents the forces due to prestress. For the calculation of the generalized inertia forces $\boldsymbol{h}_\omega$, the matrices of generalized Coriolis forces $\boldsymbol{G}_{rl}$, $\boldsymbol{G}_{el}$ and $\boldsymbol{O}_{eP}$ can be calculated from the global ansatz functions, compare e.g. [3, 9]. The external surface loads $\boldsymbol{h}_P$ follow from stress boundary conditions and the vector $\boldsymbol{h}_d$ consists of the external point forces $\boldsymbol{F}_k$ and external point moments $\boldsymbol{L}_k$. The mass, inertia, inertia coupling terms in the generalized mass matrix and the generalized Coriolis forces in Equation (12) are expressed by volume integrals, compare [9]. Some of the integrals are dependent on the elastic coordinates $\boldsymbol{q}$ which would require the evaluation of the integrals in every time step. As suggested in [11, 12, 3], the volume integrals are approximated by a Taylor-approximation and only terms that are constant or linearly dependent in $\boldsymbol{q}$ are taken into account. The quadratic terms usually play a minor role in the system dynamics and can be neglected [3]. The Taylor-approximations of the volume integrals, summarized in Table 1, have to be evaluated once before a simulation and describe together with the location of attachment points, the stiffness and damping matrices, and the elastic ansatz functions the Standard Input Data (SID), first defined in [13], which is needed to incorporate a flexible body into a multibody system model. How these terms are assembled to calculate the actual forces and inertia properties within an EMBS code is explained in detail in [3, 14, 15].

Usually, the shape functions, necessary to calculate the volume integrals from Table 1, are not available for arbitrary bodies discretized with commercial FE programs. However, the mass invariants can also be calculated from the assembled mass matrix of the free unconstrained body. For the reconstruction of the mass invariants the motion space is split in three spaces by three respectively four orthogonal projection matrices.

- First, the projection matrix which projects the motion of a free FE body to its translational motion space $\boldsymbol{S}_t$. In a pure translational motion, every node of a body moves with the same velocity so that no strain is induced in the elastic body. This motion space is spanned by the three orthogonal motions in directions of the Cartesian axis

$$\dot{\boldsymbol{q}}_t(t) = \begin{bmatrix} \vdots \\ \boldsymbol{v}(t) \\ \boldsymbol{0} \\ \vdots \end{bmatrix} = \underbrace{\begin{bmatrix} \dots \\ \boldsymbol{I} \\ \boldsymbol{0} \\ \dots \end{bmatrix}}_{\boldsymbol{S}_t} \cdot \boldsymbol{v}(t). \tag{13}$$

- Second, the projection matrix which projects the motion of the body to a pure rotational motion $\boldsymbol{S}_r$, which can be calculated by considering the velocity field of an elastic body under the three pure rotational motions $\boldsymbol{q}_r(t)$ around the Cartesian axis

$$\boldsymbol{q}_r(t) = \begin{bmatrix} \vdots \\ \tilde{\boldsymbol{R}}_{Rk}^T \cdot \boldsymbol{\omega}(t) \\ \boldsymbol{\omega}(t) \\ \vdots \end{bmatrix} = \underbrace{\begin{bmatrix} \vdots \\ \tilde{\boldsymbol{R}}_{Rk}^T \\ \boldsymbol{I}^T \\ \vdots \end{bmatrix}}_{\boldsymbol{S}_r} \cdot \boldsymbol{\omega}(t). \tag{14}$$

- Third, the projection matrix $\boldsymbol{S}_e$ which projects the motion of the free elastic body to the motion of an elastic body subject to boundary conditions. If the elastic degrees of freedom are not reduced, the elastic projector is defined as the matrix $\boldsymbol{S}_e = \boldsymbol{T}_{BC}$, where $\boldsymbol{T}_{BC}$ is the matrix which projects the free body to the constrained body due to boundary conditions. It is important to mention, that for orthogonal reduction methods no distinction $\boldsymbol{S}_{el} = \boldsymbol{S}_{er}$ between left $\boldsymbol{S}_{el}$ and right elastic projection matrix $\boldsymbol{S}_{er}$ is drawn. However, for oblique reduction techniques the left $\boldsymbol{S}_{el}$ projection matrix is different to the right projection matrix $\boldsymbol{S}_{er}$.

How the mass invariants are calculated with the four projection matrices is expressed in Table 1 where the asym operator extracts the asymmetric part of a matrix and Matlab notation $(a, b)$ is used to express the fact that only a part of a vector/matrix is calculated or used respectively.

Table 1: Elementary volume integrals

| name | definition | calculation rule |
|---|---|---|
| $m\boldsymbol{I}$ | $\int_{\mathcal{K}_0} \boldsymbol{I}\, dm$ | $(\boldsymbol{S}_t)^T \cdot \boldsymbol{M}_e \cdot \boldsymbol{S}_t$ |
| $m\boldsymbol{c}_0$ | $\int_{\mathcal{K}_0} \boldsymbol{R}_{RP}\, dm$ | $(\boldsymbol{S}_r)^T \cdot \boldsymbol{M}_e \cdot \boldsymbol{S}_t$ |
| $\boldsymbol{J}_0$ | $\int_{\mathcal{K}_0} \tilde{\boldsymbol{R}}_{RP} \cdot \tilde{\boldsymbol{R}}_{RP}^T\, dm$ | $(\boldsymbol{S}_r)^T \cdot \boldsymbol{M}_e \cdot \boldsymbol{S}_r$ |
| $\boldsymbol{C1}$ | $\int_{\mathcal{K}_0} \boldsymbol{\Phi}_P\, dm$ | $(\boldsymbol{S}_t)^T \cdot \boldsymbol{M}_e \cdot \boldsymbol{S}_{er}$ |
| ${}^W\boldsymbol{C1}$ | $\int_{\mathcal{K}_0} \boldsymbol{\Phi}_P\, dm$ | $\boldsymbol{S}_{el}^T \cdot \boldsymbol{M}_e^T \cdot (\boldsymbol{S}_t)$ |
| $\boldsymbol{C2}$ | $\int_{\mathcal{K}_0} \tilde{\boldsymbol{R}}_{RP} \cdot \boldsymbol{\Phi}_P\, dm$ | $(\boldsymbol{S}_r)^T \cdot \boldsymbol{M}_e \cdot \boldsymbol{S}_{er}$ |
| ${}^W\boldsymbol{C2}$ | $\int_{\mathcal{K}_0} \boldsymbol{\Phi}_P\, dm$ | $\boldsymbol{S}_{el}^T \cdot \boldsymbol{M}_e^T \cdot (\boldsymbol{S}_r)$ |
| $\boldsymbol{C3}$ | $\int_{\mathcal{K}_0} (\boldsymbol{\Phi}_P)^T \cdot \boldsymbol{\Phi}_P\, dm$ | $(\boldsymbol{S}_{el})^T \cdot \boldsymbol{M}_e \cdot \boldsymbol{S}_{er}$ |
| $\boldsymbol{C4}_k$ | $\int_{\mathcal{K}_0} \tilde{\boldsymbol{R}}_{RP} \cdot \tilde{\boldsymbol{\Phi}}^{P_i}_{(:,k)}\, dm$ | $\boldsymbol{C4}_{k(:,\alpha)} = -(\boldsymbol{S}_r)^T \cdot \operatorname{asym}(\operatorname{diag}(\tilde{\boldsymbol{e}}_\alpha) \cdot \boldsymbol{M}_e) \cdot \boldsymbol{S}_{er(:,k)}$ |
| $\boldsymbol{C5}_k$ | $\int_{\mathcal{K}_0} \tilde{\boldsymbol{\Phi}}_{P(:,k)} \cdot \boldsymbol{\Phi}_P\, dm$ | $\boldsymbol{C5}_k(\alpha,:) = -\boldsymbol{S}_{el(:,k)} \cdot \operatorname{asym}(\operatorname{diag}(\tilde{\boldsymbol{e}}_\alpha) \cdot \boldsymbol{M}_e) \cdot \boldsymbol{S}_{er}$ |
| $\boldsymbol{C6}_{kl}$ | $\int_{\mathcal{K}_0} \tilde{\boldsymbol{\Phi}}_{P(:,k)} \cdot \tilde{\boldsymbol{\Phi}}_{P(:,l)}\, dm$ | $\boldsymbol{C6}_{kl(\alpha,\beta)} \approx \boldsymbol{S}_{el(:,k)} \cdot \operatorname{diag}(\tilde{\boldsymbol{e}}_\alpha) \cdot \boldsymbol{M}_e \cdot \operatorname{diag}(\tilde{\boldsymbol{e}}_\beta) \cdot \boldsymbol{S}_{er(:,l)}$ |

## References

[1] Fehr, J.: Automated and Error-Controlled Model Reduction in Elastic Multibody Systems. Dissertation, Schriften aus dem Institut für Technische und Numerische Mechanik der Universität Stuttgart, Vol. 21. Aachen: Shaker Verlag, 2011.

[2] Schiehlen, W.; Eberhard, P.: Technische Dynamik – Modelle für Regelung und Simulation (in German). Wiesbaden: Teubner, 2004.

[3] Schwertassek, R.; Wallrapp, O.: Dynamik flexibler Mehrkörpersysteme (in German). Braunschweig: Vieweg, 1999.

[4] Altenbach, J.; Altenbach, H.: Einführung in die Kontinuumsmechanik (in German). Stuttgart: B.G. Teubner, 1994.

[5] Becker, E.; Bürger, W.: Kontinuumsmechanik (in German). Stuttgart: Teubner, 1975.

[6] Sokol, P.: Kinematik und Dynamik von Mehrkörperschleifen mit elastischen Körpern (in German). Dissertation. Duisburg: Universität-Gesamthochschule-Duisburg, 1990.

[7] Pascal, M.: Some Open Problems in Dynamic Analysis of Flexible Multibody Systems. Multibody System Dynamics, Vol. 5, pp. 315–334, 2001.

[8] Kübler, L.: Simulation und Sensitivitätsanalyse flexibler Mehrkörpersysteme mit großen Deformationen. Dissertation, Schriften aus dem Institut für Technische und Numerische Mechanik der Universität Stuttgart, Band 1. Aachen: Shaker Verlag, 2005.

[9] Lehner, M.: Modellreduktion in elastischen Mehrkörpersystemen (in German). Dissertation, Schriften aus dem Institut für Technische und Numerische Mechanik der Universität Stuttgart, Vol. 10. Aachen: Shaker Verlag, 2007.

[10] Simeon, B.: Numerische Simulation gekoppelter Systeme von partiellen und differential-algebraischen Gleichungen in der Mehrkörperdynamik (in German). VDI Fortschritt-Berichte, Reihe 20, Nr. 325. Düsseldorf: VDI Verlag, 2000.

[11] Melzer, F.: Symbolisch-numerische Modellierung elastischer Mehrkörpersysteme mit Anwendung auf rechnerische Lebensdauervorhersagen (in German). VDI Fortschritt-Berichte, Reihe 20, Nr. 227. Düsseldorf: VDI-Verlag, 1994.

[12] Sachau, D.: Berücksichtigung von flexiblen Körpern und Fügestellen in Mehrkörpersystemen zur Simulation aktiver Raumfahrtstrukturen. Dissertation, Universität Stuttgart, Institut A für Mechanik, 1996.

[13] Wallrapp, O.: Standardization of Flexible Body Modeling in Multibody System Codes, Part I: Definition of Standard Input Data. Mechanics of Structures and Machines, Vol. 22, No. 3, pp. 283–304, 1994.

[14] Rulka, W.: Effiziente Simulation der Dynamik mechatronischer Systeme für industrielle Anwendungen, (in German). Ph.D. thesis, Vienna University of Technology, 1998.

[15] Simpack: Simpack, Reference Guide, Simpack Version 8.901b. Simpack AG, Gilching, 2009.