Content of the Lecture

Introduction

Nowadays, the use of lighter structures demands for the consideration of elastic effects for many components. Mostly, the modeling and numerical simulation of elastic bodies is achieved with the finite element method (FEM), which often leads to a high number of degrees of freedom, more than a million degrees of freedom is not unusual. Therefore, one essential step for an efficient simulation is the reduction of the elastic degrees of freedom. The complex model is replaced with a far simpler one, where the most dominant features, input-output behavior, passivity, stability etc. of the large scale system are retained by the simpler one.

Literature Survey

Model reduction (MOR) is a very important topic for the efficient simulation and optimization of dynamical systems. The main idea of model reduction is the approximation of a large scale dynamical system (many degrees of freedom) with a system of much smaller dimension. In the last years, tremendous efforts and advances were achieved within the field of model reduction and is now a well established field in mathematics and engineering. The very nice textbook of Prof. Antoulas [1] gives a good overview and is the basis of this course.

Model reduction is applied in more and more technical applications like model reduction of thermomechanical systems [2], mechanical systems [3], electromagnetic field simulation [4], fluid dynamics [5], biomechanics [6] etc.

Model reduction is very important in multi-physics simulations where different physical domains are coupled with each other. For the mechanical part of the multi-physics simulation, reduced elastic bodies are very often an efficient choice. Typically the elastic multibody simulation method is used, where rigid and elastic bodies are coupled via ideal joints and force elements with each other and the surrounding. Model reduction in elastic multibody systems is treated e.g. in [7, 8, 9].

Basics for the description of reduced elastic bodies are linear finite element models. Model reduction for linear finite element models are typically done with the Component Mode Synthesis (CMS) [10], a modal based reduction approach. Other reduction techniques can also be used, see e.g [11].

The three main categories used for model reduction of linear systems are: modal based reduction approaches, reduction by moment-matching (implicitly achieved by projection on Krylov subspaces) and SVD-based model reduction (reduction based on Gramian matrices). With advanced methods from linear algebra, e.g. low rank approximations [12, 13], model reduction is now also possible for very large scale systems, as shown e.g. in [14].

Trends in the field are the reduction of nonlinear systems, application of MOR for mechanical models used for advanced material modeling [15, 16], model reduction from data [17] and parametric model reduction.

Additional information regarding MOR can also be found in the MOR-Wiki [18]. Currently, there is a European Model Reduction Network (EU-MORNET) [19], which brings together all major groups in Europe working on a range of model reduction strategies with applications in many domains of science and technology.

For an overview about methods, application as well as evolving problems and question in linear model reduction see e.g. [1, 20, 21, 22] and for nonlinear model reduction [23]. Links to additional papers will be uploaded in the course of the lecture.

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