Exercise Sheet 2

Exercise 3: Sparse Matrices, Static and Transient Problem of Governor

The goal of this exercise is to import the mass matrix M and stiffness matrix K of the governor discussed in the last lecture, to solve the static and transient problem and finally to reduce it with modal reduction.

(a) The mass and stiffness matrices were exported from Ansys in the Harwell-Boeing format for sparse matrices. Write a function to import the matrices located in the files `M_L.txt` and `K_L.txt`. You don't have to implement the complete specification[1] but enough to take care of these two files. Hints:

- The Matlab functions `textscan` and `fscanf` can be useful.

- Indices are counted starting from 1.

- The files also contain a right-hand side which can be ignored by your function.

- Keep in mind that the matrices are symmetric.

- Compare your results with the already imported matrices located in the files `M.mat` and `K.mat`. Use the function `load` to import them into Matlab.

(b) Plot the sparsity pattern of the matrices with the Matlab function `spy`.

(c) Apply the same constraints to the system as in the Ansys example from the lecture, i.e. Dirichlet conditions in all directions at nodes $27, 28, 29, 48, 49, 50$. Use the following table to find the corresponding degrees of freedom in the displacement vector u:

node	27	28	29	47	48	49	50	51
x	502	505	535	64	532	499	496	4
y	503	506	536	65	533	500	497	5
z	504	507	537	66	534	501	498	6

Example: The displacement in y-direction of node 48 is at index 533 of u.

(d) Solve the static problem $K \cdot u = f$ with a force f applied on nodes 47 and 51 in y-direction with 100 N each. The imported mechanical system uses the SI unit system. Solve the system of linear equations once with full matrices and once with sparse matrices. Which method is used each time by Matlab? What numerical libraries are used by Matlab each time?

(e) Solve the transient problem

$$M \cdot \ddot{u}(t) + K \cdot u(t) = f(t)$$

with f a 1 Hz sinus oscillation on nodes 47 and 51 in y-direction with an amplitude of 0.01 N in the time domain $[0, 2]$. Plot the displacement in x-direction of node 47 over time and compare it with the plot `full_transient_node_47.png` provided by us.

---

[1]You will find information about this format at `http://people.sc.fsu.edu/~jburkardt/data/hb/hb.html` and [**?**] provided in ILIAS. The rest is reverse engineering of the example on the German Wikipedia: `https://de.wikipedia.org/wiki/Harwell-Boeing-Format`

Institut für Techn. und Num. Mechanik
Profs. Eberhard / Hanss / Fehr

Model Reduction of Mechanical Systems
Prof. Dr.-Ing. J. Fehr WS 2024/25 A 2.2

First, you will have to transform this second order ODE into a first order ODE (c.f. your math lecture in the 3rd semester). You can now solve this ODE with the Matlab solver `ode23s` which expects a system of the form $y' = f(t, y)$. Don't forget to set the mass matrix and the Jacobian $\frac{\partial f}{\partial y}$ (very easy to calculate) in the options of the solver.

(f) We provide you with a reduction matrix $\boldsymbol{V}$ containing the first 10 eigenmodes[2] of the constraint system. The reduced system is built with a Galerkin projection:

$$\underbrace{\boldsymbol{V}^T \boldsymbol{M} \boldsymbol{V}}_{=:\mathbf{M}^r} \cdot \ddot{\bar{\boldsymbol{u}}}(t) + \underbrace{\boldsymbol{V}^T \boldsymbol{K} \boldsymbol{V}}_{=:\mathbf{K}^r} \cdot \bar{\boldsymbol{u}}(t) = \boldsymbol{V}^T \boldsymbol{f}(t)$$

with the reduced mass and stiffness matrices $\boldsymbol{M}^r$, $\boldsymbol{K}^r$ and reduced coordinates $\bar{\boldsymbol{u}}$. Solve the reduced system and plot as you did in (e).

(g) Compare the full and reduced solution by a reasonable method of your choice.

Notes

- The homework is due on Sunday night December 1rd, 2024 at 11:59 p.m.

- All files and partial solutions can be found on ILIAS.

---

[2]If you are interested how we calculated them: The eigenvectors corresponding to the 10 lowest eigenvalues λ of the generalized eigenvalue problem $\boldsymbol{K}\varphi = \lambda \cdot \boldsymbol{M}\varphi$ form the columns of $\boldsymbol{V}$.