Exercise Sheet 3

Exercise 4: System Response of Original and Reduced System

Consider the following dynamical system

$$\begin{aligned} \boldsymbol{M}_e \cdot \ddot{\boldsymbol{q}}(t) + \boldsymbol{D}_e \cdot \dot{\boldsymbol{q}}(t) + \boldsymbol{K}_e \cdot \boldsymbol{q}(t) &= \boldsymbol{B}_e \cdot \boldsymbol{u}(t) \\ \boldsymbol{y}(t) &= \boldsymbol{C}_e \cdot \boldsymbol{q}(t) \end{aligned}$$

and the corresponding Petrov-Galerkin[1] reduced system with the projection matrix $\boldsymbol{V}$.

We provide you with unfinished code (`exercise04.m`) and all matrices (`exercise04_matrices.mat`). Add the following features to the unfinished code to complete the comparison of the original and reduced system:

(a) Write the function `calculateSystemResponse` with the following input: matrices `Me`, `De`, `Ke`, `Be`, `Ce` and an array of imaginary points `s` where the transfer matrix should be calculated. The first output is the transfer matrix `H_N` calculated at the points `s` and the second output `H_N_norm` is the Frobenius norm of each transfer matrix.

The 7th input argument is an optional projection matrix `V`. If it is given, the transfer matrix of the corresponding Petrov-Galerkin reduced system is calculated.

(b) Calculate the relative error $\epsilon_{\mathrm{F}}^{\mathrm{rel}}$ in the Frobenius norm and plot it in a new figure with a linear scale for the abscissa and a logarithmic scale for the ordinate in the range 1 Hz to 10 kHz.

(c) Calculate the $\mathcal{H}_2$ and $\mathcal{H}_\infty$ norm of the error system. You can use the property $\boldsymbol{H}(-i\omega) = \overline{\boldsymbol{H}(i\omega)}$ which implies $\|\boldsymbol{H}(-i\omega)\|_{\mathrm{F}} = \|\boldsymbol{H}(i\omega)\|_{\mathrm{F}}$ and $\|\boldsymbol{H}(-i\omega)\|_2 = \|\boldsymbol{H}(i\omega)\|_2$. Therefore, you only need the already calculated transfer matrices since $f_{\max} = 10\,\mathrm{kHz}$ is assumed to be almost infinity.

Exercise 5: Helix

Consider a helix parameterized by

$$\boldsymbol{h}(t) = \begin{pmatrix} \cos(t) \\ \sin(t) \\ t \end{pmatrix}.$$

After projecting it with the (unknown) oblique projection matrix $\boldsymbol{P} \in \mathbb{R}^{3\times 3}$, you will get a new curve

$$\boldsymbol{x}(t) = \boldsymbol{P} \cdot \boldsymbol{h}(t).$$

Figure `exercise05.fig` shows this curve $\boldsymbol{x}(t)$. What is the matrix $\boldsymbol{P}$ and the domain of t? Describe in a few sentences how you came up with this educated guess.

<u>Notes</u>

- The homework is due on Sunday night December 11th, 2022 at 11:59 p.m.
- All files can be found on ILIAS.

[1] c.f. exercise 3 (f)